Vessel Enhancement

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Vessel Enhancement

Goal of vessel enhancement

- Single representation
- Handle degradations.
- Boost tubular structures

Vessel enhanced subimages
Vessel Enhancement Method

Given an image \( f(x) \), where \( x = \{x, y\} \), the Scale space representation is given by

\[
L(x, \sigma) = g(x, \sigma) \ast f(x)
\]

Hessian matrix

\[
H(x, \sigma) = \begin{bmatrix}
L_{xx}(x, \sigma) & L_{xy}(x, \sigma) \\
L_{yx}(x, \sigma) & L_{yy}(x, \sigma)
\end{bmatrix}
\]

The eigen values \((\lambda_1, \lambda_2)\) and eigen vectors \((v_1, v_2)\) principle curvatures and their orientations.
## Structures Corresponding to Eigenvalues

<table>
<thead>
<tr>
<th>2D</th>
<th>orientation pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>$\lambda_2$</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>L</td>
<td>H-</td>
</tr>
<tr>
<td>L</td>
<td>H+</td>
</tr>
<tr>
<td>H-</td>
<td>H-</td>
</tr>
<tr>
<td>H+</td>
<td>H+</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$|\lambda_1| \lesssim |\lambda_2|$
Vessenessness for 2D Images

\[ V_o(s) = \begin{cases} 
0 & \text{if } \lambda_2 > 0, \\
\exp\left(-\frac{R_B^2}{2\beta^2}\right)(1 - \exp\left(-\frac{s^2}{2\sigma^2}\right)) & \text{otherwise}
\end{cases} \]

Deviation to blob-like structure: \[ R_B = \frac{\lambda_1}{\lambda_2} \]

Second-order structureness: \[ S = \|H\|_F = \sqrt{\sum_{j \leq D} \lambda_j^2} \]

\[ V_o(\gamma) = \max_{s_{\text{min}} \leq s \leq s_{\text{max}}} V_o(s, \gamma) \]
Vesselsness for 3D Images

Vesselsness

The second order structure is exploited for local shape properties
<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
<th><strong>Structure</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>No noticeable structure</td>
</tr>
<tr>
<td>-</td>
<td>0</td>
<td>0</td>
<td>Plate-like, bright</td>
</tr>
<tr>
<td>+</td>
<td>0</td>
<td>0</td>
<td>Plate-like, dark</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>0</td>
<td>Line-like, bright</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td>0</td>
<td>Line-like, dark</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Blob-like, bright</td>
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</tr>
</tbody>
</table>

$|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$, $\lambda_n \in \mathbb{R}$
In the definition of vesselness the three properties are combined:

\[ \mathcal{V}(x, \sigma) = \begin{cases} 0 & \lambda_1 > 0 \lor \lambda_2 > 0 \\ \left(1 - e^{-\frac{r^2}{2\sigma^2}}\right)e^{-\frac{r^2}{2\beta^2}}\left(1 - e^{-\frac{S^2}{2c^2}}\right) & \lambda_1 \leq 0 \land \lambda_2 \leq 0 \end{cases} \]

Deviation of a plate-like structure:

\[ R_A = \frac{|\lambda_2|}{|\lambda_1|}, \quad R_A \in [0, 1] \subset \mathbb{R} \]

Similarity to blob-like structure:

\[ R_B = \frac{|\lambda_3|}{\sqrt{|\lambda_1\lambda_2|}}, \quad R_B \in [0, 1] \subset \mathbb{R} \]

Frobenius norm, second-order-like structure:

\[ S = \|H[L(x)]\|_F = \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}, \quad S \in \mathbb{R}^{0+} \]
Results on Images

Abdominal MRA

- Maximum intensity projection
- No 3D information
- Overlapping organs
2D Example: DSA
Other Tube-like Structures