Definitions

**Geometric transformation**: Given two images, what is the transformation $T$ between corresponding coordinates?

**Image Warping**: Given a source image and a transformation $T$, what does the transformed output look like?
Motivations: Panorama

Obtain a wider angle view by combining multiple images.
Time-lapse Mining from Internet Photos
Motivations: Analyzing Patterns and Shapes

What is the shape of the b/w floor pattern?

The floor (enlarged)

Transformation

Automatically rectified floor

Slide from Antonio Criminisi
Motivations: Image Editing
Global Geometric Transformation

- Original
- Translation
- Rotation
- Aspect
- Scaling
- Affine
- Perspective
Global Geometric Transformation

- Transformation $T$ is a coordinate-changing machine:
  \[ p' = T(p) \]

- What does it mean that $T$ is **global**?
  - It is the same for any point $p$
  - It can be described by just a few numbers (parameters)

- Let’s represent $T$ as a matrix:
  \[
  \begin{bmatrix}
  x' \\
  y'
  \end{bmatrix}
  =
  T
  \begin{bmatrix}
  x \\
  y
  \end{bmatrix}
  =
  \begin{bmatrix}
  a_1 & a_2 \\
  a_3 & a_4
  \end{bmatrix}
  \begin{bmatrix}
  x \\
  y
  \end{bmatrix}
  +
  \begin{bmatrix}
  t_x \\
  t_y
  \end{bmatrix}
  \]

Source: Alyosha Efros
\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = T \begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
a_1 & a_2 \\
a_3 & a_4
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} + \begin{bmatrix}
t_x \\
t_y
\end{bmatrix}
\]

Translation

Rotation

Aspect Scaling

Affine

Perspective

Translation

Rotation

Aspect

Scaling

Affine

Perspective
How to Estimate Geometric Transformation

Estimate $T$ is to estimate parameters $a_1, a_2, a_3, a_4, t_x, t_y$

Three correspondences can compute a transformation

Given a set of correspondences (include both inliers and outliers)
How to derive a correct $T$?

**Inliers:** correct pairs of matching points which comply with the true transformation

**Outliers:** erroneous pairs of matching points from two images which hurt the quality of parameter estimates
Outliers affect least squares fit
Outliers affect least squares fit
RANSAC for Estimate Geometric Transformation

• **RANdom Sample Consensus**

• **Approach**: we want to avoid the impact of outliers, so let’s look for “inliers”, and use those only.

• **Intuition**: if an outlier is chosen to compute the current fit, then the resulting line won’t have much support from rest of the points.
RANSAC for line fitting example

Source: R. Raguram

Lana Lazebnik
RANSAC for line fitting example

Least-squares fit

Source: R. Raguram
RANSAC for line fitting example

1. Randomly select minimal subset of points

Source: R. Raguram

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1. Randomly select minimal subset of points
2. Hypothesize a model

Source: R. Raguram

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RANSAC for line fitting example

1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function

Source: R. Raguram
RANSAC for line fitting example

1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model

Source: R. Raguram
RANSAC for line fitting example

1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat hypothesize-and-verify loop

Source: R. Raguram
RANSAC for line fitting example

1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram
RANSAC for line fitting example

1. Randomly select minimal subset of points
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Uncontaminated sample

Source: R. Raguram
RANSAC for line fitting example

1. Randomly select minimal subset of points
2. Hypothesize a model
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5. Repeat hypothesize-and-verify loop

Source: R. Raguram

Lana Lazebnik
RANSAC for line fitting

- Repeat $N$ times:
  - Draw $s$ points uniformly at random
  - Fit line to these $s$ points
  - Find inliers to this line among the remaining points (i.e., points whose distance from the line is less than $t$)
  - If there are $d$ or more inliers, accept the line and refit using all inliers
• That is an example fitting a model
• (line)...

• What about fitting a transformation (translation)?
RANSAC example: Translation

Select *one* match, count *inliers*
RANSAC example: Translation

Select *one* match, count *inliers*
RANSAC example: Translation

Find “average” translation vector
RANSAC for estimating homography

• RANSAC loop:
  • 1. Select four feature pairs (at random)
  • 2. Compute homography $H$ (exact)
  • 3. Compute inliers where $SSD(p_i', Hp_i) < \varepsilon$
  • 4. Keep largest set of inliers
  • 5. Re-compute least-squares $H$ estimate on all of the inliers

Slide credit: Steve Seitz
RANSAC pros and cons

• Pros
  – Simple and general
  – Applicable to many different problems
  – Often works well in practice

• Cons
  – Lots of parameters to tune
  – Doesn’t work well for low inlier ratios (or can fail completely)
  – Can’t always get a good initialization of the model based on the minimum number of samples
Recent Work for Geometric Transformation based on Correspondences


Mosaics

Obtain a wider angle view by combining multiple images.
Image warping

Given a coordinate transform $T$ and a source image $f(x,y)$, how do we compute a transformed image $g(x',y') = f(T(x,y))$?
Image warping

- Send each pixel $f(x,y)$ to its corresponding location $(x',y') = T(x,y)$ in the second image.

Q: what if pixel lands “between” two pixels?
Image warping

- Send each pixel $f(x,y)$ to its corresponding location $(x',y') = T(x,y)$ in the second image

Q: what if pixel lands “between” two pixels?

A: Interpolate color value from neighbors
  - nearest neighbor, bilinear...

Slide from Alyosha Efros, CMU
Bilinear interpolation

Sampling at $f(x,y)$:

\[
\begin{align*}
(i,j) & \quad (i+1,j) \\
(i,j+1) & \quad (i+1,j+1)
\end{align*}
\]

\[f(x,y) = (1-a)(1-b) \quad f[i,j] + a(1-b) \quad f[i+1,j] + ab \quad f[i+1,j+1] + (1-a)b \quad f[i,j+1]\]
Nearest Neighbor (3X)

HQ 3X http://en.wikipedia.org/wiki/Image_scaling
Recap: How to stitch together a panorama (a.k.a. mosaic)?

• Basic Procedure
  – Take a sequence of images from the same position
    • Rotate the camera about its optical center
  – Compute transformation (homography) between second image and first using corresponding points.
  – Transform the second image to overlap with the first.
  – Blend the two together to create a mosaic.
  – (If there are more images, repeat)

Source: Steve Seitz
Robust feature-based alignment

Source: L. Lazebnik
Robust feature-based alignment

- Extract features

Source: L. Lazebnik
Robust feature-based alignment

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  - Hypothesize transformation $T$ (small group of putative matches that are related by $T$)

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  – *Verify* transformation (search for other matches consistent with $T$)

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Robust feature-based alignment

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- Loop:
  - *Hypothesize* transformation $T$ (small group of putative matches that are related by $T$)
  - *Verify* transformation (search for other matches consistent with $T$)

Source: L. Lazebnik
Creating and Exploring a Large Photorealistic Virtual Space

Josef Sivic, Biliana Kaneva, Antonio Torralba, Shai Avidan and William T. Freeman,
Internet Vision Workshop, CVPR 2008.
http://www.youtube.com/watch?v=E0rboU10rPo
Summary: transformation & warping

• Write **2d transformations** as matrix-vector multiplication

• **Estimating geometric transformations**: solve for unknown parameters given corresponding points from two views (exemplar method: RANSAC).

• **Mosaics**: uses homography and image warping to merge views taken from same center of projection